Sheet (6)

3.1. Find the Laplace transform of

(a) $x(t) = -e^{-at}u(-t)$

(b) $x(t) = e^{at}u(-t)$

3.5. Find the Laplace transform $X(s)$ and sketch the pole-zero plot with the ROC for the following signals $x(t)$:

(a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

(b) $x(t) = e^{-3t}u(t) + e^{2t}u(-t)$

(c) $x(t) = e^{2t}u(t) + e^{-3t}u(-t)$

3.6. Let

$$x(t) = e^{-a|t|}$$

Find $X(s)$ and sketch the zero-pole plot and the ROC for $a > 0$ and $a < 0$.

3.16. Find the inverse Laplace transform of the following $X(s)$:

(a) $X(s) = \dfrac{1}{s+1}, \ \mathrm{Re}(s) > -1$

(b) $X(s) = \dfrac{1}{s+1}, \ \mathrm{Re}(s) < -1$

(c) $X(s) = \dfrac{s}{s^2+4}, \ \mathrm{Re}(s) > 0$

(d) $X(s) = \dfrac{s+1}{(s+1)^2+4}, \ \mathrm{Re}(s) > -1$

3.17. Find the inverse Laplace transform of the following $X(s)$:

(a) $X(s) = \dfrac{2s+4}{s^2+4s+3}, \ \mathrm{Re}(s) > -1$

(b) $X(s) = \dfrac{2s+4}{s^2+4s+3}, \ \mathrm{Re}(s) < -3$

(c) $X(s) = \dfrac{2s+4}{s^2+4s+3}, \ -3 < \mathrm{Re}(s) < -1$

Sheet (6)

3.25. The output $y(t)$ of a continuous-time LTI system is found to be $2e^{-3t}u(t)$ when the input $x(t)$ is $u(t)$.

(*a*) Find the impulse response $h(t)$ of the system.

(*b*) Find the output $y(t)$ when the input $x(t)$ is $e^{-t}u(t)$.

3.30. Consider a continuous-time LTI system for which the input $x(t)$ and output $y(t)$ are related by

$$y''(t) + y'(t) - 2y(t) = x(t) \tag{3.86}$$

(*a*) Find the system function $H(s)$.

(*b*) Determine the impulse response $h(t)$ for each of the following three cases: (*i*) the system is causal, (*ii*) the system is stable, (*iii*) the system is neither causal nor stable.